Analysis and Probability in Infinite Dimensions

Bad Herrenalb, Germany

22-25 April 2025

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Abstracts of Talks

Tuesday, April 22

Stability of C_0-semigroups and analytic Besov functions

Charles Batty

University of Oxford

This talk will cover a selection of results about stability of C_0-semigroups which depend on the spectral properties of the generator, starting from the 1980s and coming up to recent results (if time permits). Some of the results are well known and some less so; I was involved with some, but not all, of them. Analytic Besov functions play important roles in some of the recent results.

An overview of numerical schemes for stochastic hydrodynamics

Hakima Bessaih

Florida International University

Stochastic hydrodynamics plays a fundamental role in modeling fluid systems subject to random perturbations, with applications ranging from turbulence modeling to biological flows and climate dynamics. The mathematical framework often involves stochastic partial differential equations (SPDEs), such as the stochastic Navier-Stokes equations, which introduce significant analytical and computational challenges.

In this talk, I will present an overview of numerical schemes designed for stochastic hydrodynamic models. Special attention will be given to temporal integration strategies, such as explicit, implicit, and semi-implicit methods, as well as splitting methods.

Transference for harmonic oscillators

Pierre Portal

Australian National University

Transference is a powerful technique that exploits common algebraic structures in order to transfer analytic results from one setting to another. In its simplest form, it reduces questions about the functional calculus of a group generator acting on a Banach space X to questions about Fourier multipliers on $L^p(X)$. This uses the Fourier representation of the commutative group $\mathbb{R}$. When dealing with harmonic oscillators such as $\Delta - |x|^2$ or the Ornstein-Uhlenbeck operator, transference reaches its full potential when using the more appropriate Schrödinger representation of the Heisenberg group. More generally, I claim that operators that have the same algebraic structure as sublaplacians on Lie groups are well suited to transference methods. In this talk, I aim to demonstrate this principle through a few highlights of the work I have done with Jan van Neerven and other collaborators on the topic. This includes substantial simplifications of classical proofs (by exploiting the "right" algebraic structure), mathematical physics consequences (constructing time as an unsharp observable), and the resolution of problems in non-commutative geometry using their algebraically analogue counterpart in Banach space-valued harmonic analysis.

Functional calculus, square functions, and representation formulas

Bernhard Haak

Université de Bordeaux

Representation formulas are foundational to the very construction of functional calculi, reflecting the idea that applying a function to an operator is generally realized through expressing the function in a form amenable to operator substitution. It has long been known that square function estimates are closely connected to the boundedness of H^∞ functional calculus in various settings. A particularly elegant route to this connection is, again, through representation formulas, and some operator theory. We illustrate this perspective with several examples drawn from the literature and contrast it with some of the current frontiers and open questions. This talk relies in great part on ongoing work with Markus Haase.

Evolutionary semigroups on the path space

Markus Kunze

Universität Konstanz

We introduce the notion of an ‘evolutionary semigroup’ on the path space, which can be used to generalize the concept of ‘transition semigroup’ to certain stochastic processes which are not necessarily Markovian. We show that the transition semigroup of any Markov process (that has the right path-regularity) can be lifted to an evolutionary semigroup. Other examples include solutions to (stochastic) delay equations. As a first application, we show how martingales can be represented by means of the evolutionary semigroup. In particular, our results show that martingales are mild solutions of certain final value problems involving the generator of the evolutionary semigroup. This is a new result even in the Markovian setting.

Wednesday, April 23

Semigroups generated by elliptic operators with operator-valued coefficients

Wolfgang Arendt

Universität Ulm

Systems of elliptic operators do not enjoy all the good properties known for a single elliptic operator. Still there are some interesting positive results. Using an elegant criterion due to Robin Nittka we will show that the semigroup which is generated on L^2 extends to L^p for p in a certain interval around 2. We will do this not only for systems but suppose right away that the coefficients of the elliptic operators take their values in the space of all bounded operators on a Hilbert space H as it is also done in the literature.

If this space H is an L^2 space one may ask oneself under which conditions the semigroup is positive. In the scalar case this is always true. For systems the answer will be given during the talk, which is based on joint work with Tom ter Elst (Auckland) and Manfred Sauter (Ulm).

On the notion of pathwise weak solutions for parabolic stochastic PDEs

Pascal Auscher

Université Paris-Saclay

We explain a new notion of solutions for parabolic stochastic PDEs in divergence form driven by a cylindrical Brownian with non-smooth elliptic coefficients, L^p data, $1 < p < \infty$ and forcing terms, where trajectories are understood after testing against test functions and not in the usual sense of processes being almost surely in a space of continuous Banach space-valued functions.

Path spaces use tent spaces which apply well despite the lack of smoothness of coefficients. The ansatz to build solutions is a reduction to a perturbed stochastic heat equation. The needed estimates rely on tent space extrapolation for two deterministic maximal regularity operators: the Lions operator and a quadratic operator coming from use of the Itô formula for stochastic heat equations. This induces existence and uniqueness of pathwise solutions.

This is joint work with Pierre Portal.

Well-posedness of magnetic evolution equations on adapted modulation spaces

Dorothee Frey

Karlsruher Institut für Technologie

In this talk, we study wave and Schrödinger equations for magnetic Schrödinger operators with unbounded background fields. Based on a magnetic phase space transform, we construct a parametrix for such operators, and establish well-posedness results in modulation spaces adapted to the magnetic potential.

This talk is based on joint work with S. Weng.

The role of interpolation theory in the study of parabolic boundary value problems

Emiel Lorist

Technische Universiteit Delft

In the analysis of deterministic and stochastic evolution equations, the real interpolation method plays a key role in identifying the optimal function space for the initial value u_0 in the context of maximal L^p-regularity of the abstract Cauchy problem

$$\begin{cases} u_t + Au & = f, \\ u(0) & = u_0. \end{cases}$$

Here, u_0 can be interpreted as an inhomogeneous boundary condition at the temporal boundary $t = 0$. Now, consider the case where A is an elliptic differential operator defined on a domain $\mathcal{O}$. By analogy with the temporal boundary, one might expect that the optimal function space for an inhomogeneous boundary condition at the spatial boundary $\partial\mathcal{O}$ would also be given as an interpolation space. However, classical interpolation methods are not able to fully describe the optimal spaces for spatial inhomogeneities. In this talk, I introduce a novel, discrete interpolation framework for Banach spaces. I will discuss how this framework extends and unifies various results in the literature and, most importantly, discuss its role in the study of parabolic boundary value problems.

This talk is based on joint work with Nick Lindemulder.

Thursday, April 24

Weak martingale solutions to stochastic Navier-Stokes-Cahn-Hilliard system with transport noise

Zdzisław Brzeźniak

University of York

In this talk, we investigate the weak solvability of an initial boundary value problem known as the Navier-Stokes-Cahn-Hilliard system, which describes the dynamics of a homogeneous, incompressible and isothermal mixture of two immiscible Newtonian fluids flowing in a bounded 2D or 3D domain under stochastic perturbations.

We assume that the density and viscosity of the mixture are constants and, to prove the existence result, we consider an approximation problem and use the Jakubowski-Skorohod Theorem to prove that the laws of the corresponding solutions on a certain non-metric topological space Z_T have a sequence weakly convergent to a new probability measure on Z_T.

Now, by following the argument of Mikulevicius and Rozovskii in their paper (Ann. Probab. 33(1) (2005), 137–176) with some modifications, we prove that the canonical process on the space Z_T is in fact a martingale solution of our problem with respect to the new measure.

The approach is quite interesting compared to the existence approach in the literature, since we combine both the Jakubowski-Skorohod theorem and the Mikulevicius and Rozovskii argument to deal with our problem.

This talk is based on a joint research with Aristide Ndongmo Ngana (York).

A geometric dichotomy for the discrete Hilbert transform

Tuomas Hytönen

Aalto University

I will report on joint work with Assaf Naor in which we identify the rate of growth of the norms of finite Hilbert transforms as a new Banach space parameter ranging continuously from 0 for UMD (unconditional martingale differences) spaces to 1 for non-super-reflexive spaces. All intermediate rates are shown to be attained by explicitly constructed spaces that are necessarily super-reflexive without UMD. With this tool, we also obtain precise quantitative information about the amount of deviation of martingale type from Rademacher type in classical examples of Pisier (1975), where only upper and lower estimates were previously available. With the help of the finite Hilbert transforms, we obtain efficient indirect bounds for the martingale type without having to provide any examples of martingales with extremal behaviour.

Chemotaxis with random perturbation

Erika Hausenblas

Montanuniversität Leoben

Cross-diffusion is a phenomenon that arises in various applications, such as population dynamics, ecology, biology, and material science. It describes systems with multiple interacting components or species, where the concentration gradient of one component influences the motion of another. In other words, cross-diffusion occurs when the diffusion of one substance is affected by the presence of another.

These cross-coupling effects can lead to complex and often nonlinear behaviours, including pattern formation, spatial segregation, and blow-up phenomena. Cross-diffusion plays a significant role in many physical, chemical, and biological systems, such as chemical reactions, population dynamics, and fluid mechanics. A well-known example of a cross-diffusion system is the chemotaxis model, which describes the movement of biological species in response to chemical signals.

In our talk, we will first introduce a general cross-diffusion system. We will then focus on the chemotaxis system, where the leading operator involves a porous medium-type term. Our main results concern the existence of solutions for the stochastic chemotaxis system. This work is carried out in collaboration with Debopriya Mukherjee and Ali Zacharias.

Stability via closure relations with applications to coupled partial differential equations

Birgit Jacob

Bergische Universität Wuppertal

We consider differential operators A that can be represented by means of a so-called closure relation in terms of a simpler operator A_{ext} defined on a larger space. We analyze how the spectral properties of A and A_{ext} are related and give sufficient conditions for exponential stability of the semigroup generated by A in terms of the semigroup generated by A_{ext}. As applications we study the long-term behaviour of a coupled wave-heat system on an interval, parabolic equations on bounded domains that are coupled by matrix valued potentials, and of linear infinite-dimensional port-Hamiltonian systems with dissipation on an interval.

SPDEs driven by cylindrical Lévy processes

Markus Riedle

King's College London

Cylindrical Lévy processes are generalised processes in Banach or Hilbert spaces that do not take values in the underlying space. Prominent examples include the cylindrical Brownian motion and the standard symmetric α-stable cylindrical process. While these processes are natural extensions of their finite-dimensional counterparts, they cannot be realised as classical stochastic processes attaining values in the underlying space.

In this talk, we introduce the concept of cylindrical Lévy processes and present several key examples. The main focus will be on establishing the existence and uniqueness of solutions to an abstract evolution equation in a Hilbert space driven by an arbitrary cylindrical Lévy process. Since these processes do not satisfy a semimartingale decomposition and may lack finite moments, standard methods cannot be applied to prove the desired result. We will outline the novel techniques used to address these challenges, emphasising the key ideas in the proof.

This talk is based on joint works with G. Bodo, S. Cox, and others.

Ricci curvature bounds for Markov semigroups

Melchior Wirth

Universität Leipzig

Lower Ricci curvature bounds for a Riemannian manifold have strong implications on the geometry of the manifold as well as analytic and probabilistic properties of the heat semigroup and Brownian motion. In recent years, several notions of Ricci curvature bounds have been developed for abstract Markov semigroups that apply to spaces without differential structure such as discrete graphs or even "noncommutative spaces" underlying the dynamics of open quantum systems. In this talk, I will review several Ricci curvature notions, in particular Bakry-Émery curvature, intertwining curvature and entropic curvature, with a focus on their permanence properties and applications to decay to equilibrium bounds.

This talk is partly based on joint work with Florentin Münch and Haonan Zhang.

Chasing regularization by noise of 3D Navier-Stokes equations

Antonio Agresti

Sapienza Università di Roma

Global well-posedness of 3D Navier-Stokes equations (NSEs) is one of the biggest open problems in modern mathematics. A long-standing conjecture in stochastic fluid dynamics suggests that physically motivated noise can prevent (potential) blow-up of solutions of the 3D NSEs. This phenomenon is often referred to as 'regularization by noise'. In this talk, I will review recent developments on the topic and discuss the solution to this problem in the case of the 3D NSEs with small hyperviscosity, for which the global well-posedness in the deterministic setting remains as open as for the 3D NSEs. An extension of our techniques to the case without hyperviscosity poses new challenges at the intersection of harmonic and stochastic analysis, which, if time permits, will be discussed at the end of the talk.

Friday, April 25

Dual, integrated and twin semigroups associated with delay equations

Odo Diekmann

Universiteit Utrecht

When one wants to associate a dynamical system with a delay equation, one needs to specify the underlying state space. Since there is a bit of freedom in choosing this space, various approaches coexist. The aim of this talk, based on joint work with Mats Gyllenberg and Sjoerd Verduyn Lunel, is to reveal the relationship between these approaches. The main conclusions are

- For retarded equations with finite delay the dual semigroup framework can, in a natural way, be embedded into the twin framework;

- Integrated semigroups are only needed when one extends the original state space in an unnatural/half-hearted way (but they are helpful in understanding the degeneracy that does occur for twin semigroups with multi-valued infinitesimal generators);

- The twin approach allows to develop theory for neutral equations, whereas one gets stuck when trying to do so in the dual semigroup framework.

Functional analysis of neural networks

Francesca Bartolucci

Technische Universiteit Delft

Neural Networks are a widely used mathematical model for solving problems in Machine Learning. While their success in applications is indisputable, their theoretical understanding remains limited, and Neural Networks are often used as black boxes, meaning the input-output process is unknown. The limited theoretical understanding becomes particularly delicate when using artificial intelligence in areas like medicine and security, where high reliability is required. Advancing our theoretical understanding of Neural Networks to improve their transparency remains a significant challenge for the scientific community, where mathematicians have a vital role to contribute. In this talk, we explore the functional analysis of Neural Networks, delving into the function spaces defined by these models. In particular, leveraging the theory of reproducing kernel Banach spaces, combined with variational results, we derive representer theorems that justify the finite architectures commonly employed in applications.

TBA

Lutz Weis

Karlsruher Institut für Technologie

List of Posters

Vector-valued pointwise ergodic theorems for operators
Micky Barthmann, *Technische Universität Chemnitz*

Weak L^p bounds for stochastic integrals in Banach spaces
Gergely Bodo, *Universiteit van Amsterdam*

Well-posedness of semilinear stochastic evolution equations
Katharina Klioba, *Technische Universiteit Delft*

Uniform large deviation principle for the solutions of two-dimensional stochastic Navier-Stokes equations in vorticity form
Ankit Kumar, *Montanuniversität Leoben*

Stochastic geometric PDEs
Erwin Luesink, *Universiteit van Amsterdam*

On the stochastic chemotaxis-fluids system
Boris Jidjou Moghomye, *Montanuniversität Leoben*

Markov Switching Multiple-equation Tensor Regressions
Qing Wang, *Università Ca' Foscari Venezia*

H^∞ Calculus in polyhedral domains
Tobias Werner, *Universität Kassel*

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